The Creation and Developement of Mutation Geometry.

I should like to present for the consideration of the readers of School Science and Mathematics what I consider a new mode in geometric thinking.

The content of the new mode is contained in one axiom - postulate (alpha) and in one proposition (omega) and their consequences.

If a and b are any two directed line segments(vectors) their product a.b shall be defined as a times the projection of b upon a.

Axiom - postulate (alpha)

If a and b are any two vectors constant in length and maintaining a constant angle between them then the product a.b is tempo - locally invariant. By this we mean no instant in time nor position in space can affect its value. We may toss,drag, conceive, or change a and b in any fashion from here to there and the value of the product a.b under (alpha) will remain unchanged. What a simple statement of fact. The ancients knew that or would recognize it as a fact when told to them. I should remind the reader that a and b are variables under(alpha); variables in sense.

Consequences of Tempo - Local Invariance

In space	In time
Communization of variables	Submersion of Variable
Communization Resolution	Submersion Resolution
Endless Communization	Endless Submersion

An Assemblage of Products

We often wish, and (alpha) gives us the right, to assemble our products so that all have a common line. Such a process of grouping we call a communization. We may perform such a process on an endless number of products. When such a resolution has been performed the non-common members of each product will take up certain positions, within limits, at the discretion of the operator. These non-common vectors in their new positions are called the communes of their former selves. In any analysis the operator states where his communes are.

We shall for the present deal with space communization. When we shall have proven proposition (omega) we shall illustrate the smoothe searching power of the new geometry by a few elementary problems.

Notation . We shall denote the magnitude of a directed line segment a by a_o and its unit direction by a'.

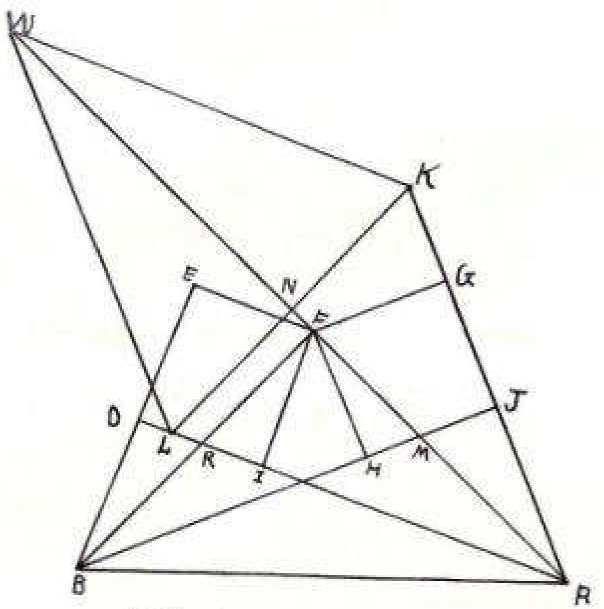


Fig. I

Proposition (omega)

A double product of the form P = a.cb.c can be reduced to the (alpha) form P = (a.b + a.d)/2. where d is the symetric of b with regard to c. It is called the transmute of b. If we have a form that is not of the form a.cb.c we can communize it untill it is of that form by (alpha). For greater ease in followg the proof we shall assume that c is a unit direction.

Argument. See Fig. I. Let any arbitrary directed line segments AB and AK be denoted by a and b respectively. Let any arbitrary direction as AR be denoted by c.

Draw a line AL equal to AK such that angle LAM equals angle KAM. Complete the rhombus AKLW. Draw Bj, BF, and BD perpendicular to AK, AW, and AL respectively. Draw FH, FG, FE, and FI perpendicular to BJ, AK, BD, and AL respectively. In rt. triangles MAJ and MBF there is a common angle at M. So angle MAJ equals angle MBF. In rt. triangles RAF and RBD there is a common angle at R. So angle RBD equals angle RAF. So angle RBD equals angle MBF. So BF is the bisector of angle EBH. In that case we have JG = HF = FE = DI. Also FI = FG and AG = AI. Now

AG = AJ + JG = AJ + DI = AJ +AD - AI or AG = AJ + AD - AG.

2AG = AJ + AD. Multiply both sides by AK and we get:

2(AK)(AG) = (AK)(AJ) + (AK)(AD).

From the similar rt. triangles AFG and ANK we have

AK/AF = AN/AG so (AK)(AG) = (AF)(AN). So

2(AF)(AN) = (AK)(AJ) + (AK)(AD). Now

AF = a.c and (AK)(AJ) = a.b

AN = b.c and (AK)(AD) = (AL)(AD) = a.d

where AL is represented by d. Then

2a.cb.c = a.b + a.d which was to be demonstrated

Consequences of Proposition (omega)

Disassociation of variables

Disassociation Resolution

Endless disolution of products

If we have a sum of products of the form

P = a.bc.d - e.fg.h and so on we may resolve them by (alpha) followed by (omega) and in the end arrive at a simple product of the form P = m.n.

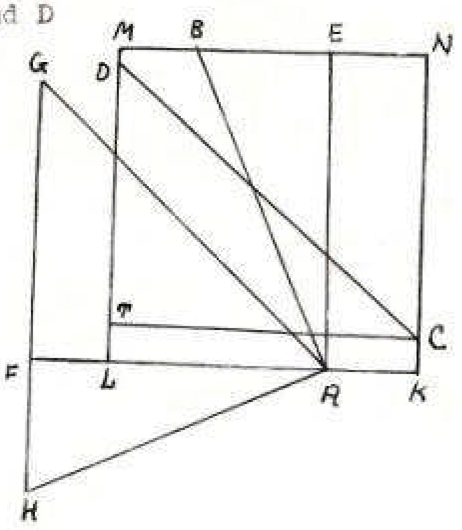
What good does the proving of (Omega) do?

The answer is: it will do practically no good unless the mutual interplay between (alpha) and (omega) is constantly born in mind. The combination resolving power of (alpha) and (omega) can not be over emphasized because it carries with it the the implication of a visual analytical resolving power that man has not heretofore enjoyed. It should supercede forever the old cumbersom way of geometric constructions.

In elementary work it is seldom ever necessary to use more than (alpha) the single product.

I shall now illustrate the new geometry by a couple of simple problems whose solutions can be gotten in various other ways.

Problem. Given four pts. A,B,C, and D to draw a square such that one side will pass thru each of the four pts.



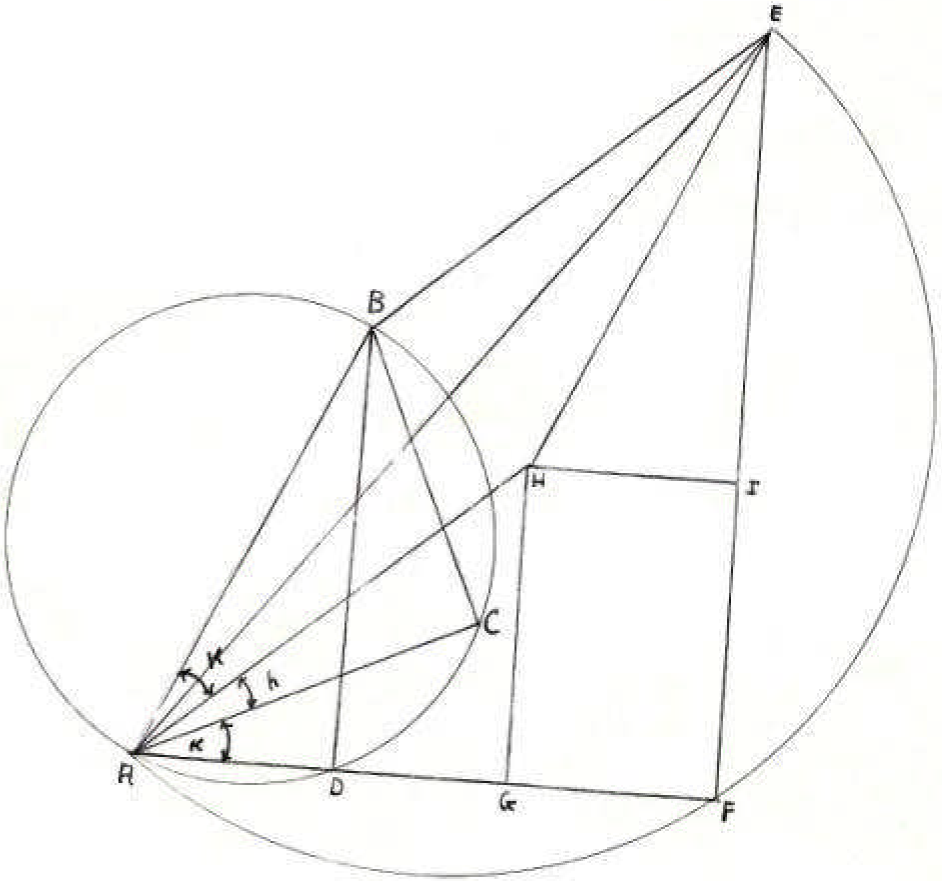
Designate the known line segments CD and AB by a and b respectively. Let s and p designate the unit directions of the sides of the square thru A and C. Then $a.s = b.p$ states the the equality between ajacent sides of the square . Now let us communize this equation and write $a.s = c.s$ where c is the commune of b. This may be written $(a-c).s = 0$. The only unknown in this equation is the unit direction s which according to the def. of multiplication is perpendicular to the known $(a-c)$ as long as it has a value. We shall now do the construction according to the directions in $(a-c).s = 0$

Draw AG parallel and equal to CD. Draw AH perpendicular and equal to AB. Draw AF perpendicular to HG. Thru B,C,and D draw lines parallel and perpendicular to AF forming the rectangle KLMN. Then KLMN is a square.

Proof. Draw AE and CT perpendicular to MN and ML. Now rt. triangles CDT and AGF are congruent having their hypoteni CD and AG equal by construction and their sides thru A and C parallel. So CT = AF. Rt. triangles ABE and AFH are congruent their hypoteni AB and AH equal by construction and their sides thru A perpendicular . So AE = AF. Therefore

CT = AE

which makes KLMN a square. Discussion . Our communized equation shows that if $a = c$ or what is the same thing that if AB and CD are equal and perpendicular,regardles of what position they may occupy on the plane that every rectangle drawn thru their end pts. is a square. We may choose A and B; A and C; or A and D as pts. on opposit sides and get corresponding squares.



Given a circle whose diameter is AB. To draw two chords thru pt. A meeting at a given angle angle K whose sum shall be equal to a given line segment m. Let the unit directions of the chords thru A be indicated by p and s. Designate AB by a. Then

a.s + a.p = m . Communizing this equation we get:

a.s + c.s = m. or

(a + c).s = m where c is the commune of a.

We shall now carry out the directions of the last equation. Draw AH equal to AB and making the given angle K with AB. Complete the parallelogram ABHE. Draw a circle on AE as diameter. With A as a center and m as a radius cut the circle AE in F. Draw AF cutting circle AB in D. Make angle DAC equal the given angle K. Then chords AD and AC are the required chords. Proof. Draw HI and HG perpendicular to EF and AF. Rt triangles ABD and HEI are congruent having their hypoteni equal and their sides parallel. So

AD = HI = GF

Rt. triangles ABC and AHG are congruent having their hypoteni AB and AH equal by construction and their acute angles = K + h. So

AC = AG. Adding the last two equations we get

AD + AC = GF + AG = AF = m.

If we want the difference of the two chords to be equal to the given line m all we have to do is to reverse AH and proceed as before.

We may look forward to many interesting and exciting victories with the new geometry.

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